# A comparison of fuzzy and CPWL approximations in the continuous-time nonlinear model-predictive control of time-delayed Wiener-type systems

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**Abstract** This paper deals with a novel method of continuous-time model-predictive control for nonlinear time-delayed systems. The problems relating to time delays are solved by incorporating the Smith-predictor scheme in a control-law derivation. A nonlinear-mapping approximation, employing either continuous piece-wise linear functions or a fuzzy system, is also an integral part of the control scheme, and thus removes the need for output-function invertibility. An illustrative experiment is conducted to compare the control quality in both approaches when tackling a time-delayed Wiener-type system control.

**Keywords** Nonlinear predictive control · Continuous systems · Time-delayed systems · Fuzzy systems · Piece-wise linear functions · Wiener-type model

## 1 Introduction

Wiener-type systems are a special class of nonlinear systems that are mainly used for modelling nonlinear processes encountered in process industries, for example, pH neutralization processes [16, 3] and distillation processes [17]. The large number of Wiener-system-based predictive-control methods developed in recent years [16, 9, 12] indicates the interest in this field in the control community. An additional difficulty is that some of the systems also incorporate pure time delays [21] as a consequence of non-ideal situations such as non-ideal mixing and transportation time delays. This fact means that some of the ”classical” approaches are unable to reach a satisfactory level of control quality.

Nonlinear-model predictive-control (NMPC) methods were initially developed as an extension of extremely successful model-predictive control (MPC) methods [8] based on linear models. The key motivation was to employ more accurate nonlinear models in process prediction and optimization, and thus achieve better control quality for highly nonlinear processes and moderately nonlinear processes with large operating regimes. For the state of the art of NMPC methods the reader is referred to the papers by Morari [15] and Henson [10]. The majority of nonlinear predictive methods are based on a discrete-time representation; however, this can result in several shortcomings in terms of relying on a system approximation and inaccurate system intersample behaviour [13]. Moreover, a description of the system in a continuous-time domain is much more natural, especially in the case of the Wiener-type systems. The first attempt to resolve the issues of continuous-time predictive control was made in the 1990s by Demircioğlu [6]. The discrete-time-based method of generalized predictive control (GPC) [5] was first of all reformulated into the continuous-time domain for the SISO [6] and MIMO [7] systems, and then extended to nonlinear systems [4]. Our proposed method relates to these works, and also attempts to resolve the following issues:

- Some discrete-time NMPC methods (e.g., [2, 19]) suffer from having to solve an on-line non-convex optimization problem, which is in general computationally expensive and can lead to solutions with local minima.
- Since the core of all model-based predictive methods is explicit-model-based optimal open-loop control [4], the model's accuracy plays a very important role. For Wiener models in particular, an inadequate nonlinear-mapping approximation can adversely affect the control quality in some operating regions.
- When dealing with time-delayed models, in the work by Demircioğlu [6] the process-model order is augmented by incorporating the Padé approximation of the time delay into the model. However, a model-order increase can lead to the bigger computational burden of a control algorithm and is, in general, avoided if possible.

The proposed approach in this paper tackles the control of time-delayed Wiener-type systems using continuous-time nonlinear model-based predictive control. The problem of pure time delays is approached by estimating the auxiliary undelayed process output and including it in the designated cost function. Thus, the original idea of the Smith predictor [20] is directly involved in the control law, and the model order does not need to be augmented. The receding-horizon strategy was combined with a cost function that, by adopting the ideas from predictive functional control [22], minimizes the difference between the future-output-prediction error and the model-prediction error. In this way the control law is closed-form optimal, and on-line optimization is not needed. The output nonlinearity of the Wiener model is approximated by using two different approaches: continuous piece-wise linear (CPWL) functions [11] and a static Takagi-Sugeno-type fuzzy system (FS) [24, 1]. The FS approximation is based on the method of robust Wiener-model identification presented by Škrjanc [23]. Unlike the methods that invert the static nonlinearity and transform the control problem to a linear one [16, 12], here the CPWL and FS approximations are a part of the calculation of the output prediction. This in fact raises the key questions of this paper - how do the approximations function in an analytical prediction of the model output and what are the effects on the closed-loop control quality?

The outline of the paper is as follows. In Section 2 the CPWL and FS functions are introduced. In Section 3 the model-output predictions for both cases are formulated in the continuous-time domain, and the nonlinear predictive control law is derived. Section 4 gives a comparison of the closed-loop-control results for both cases using a simple and illustrative example. Section 5 presents the conclusions.

## 2 Problem formulation

Let us assume a nonlinear time-delayed continuous-time system

$$\begin{aligned} \dot{x}_p(t) &= f(x_p(t), u(t)) \\ y_p(t) &= g(x_p(t-T_d)) \end{aligned} \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}$ are smooth functions, $x_p \in \mathbb{R}^n$ is a vector of $n$ state variables, $T_d$ denotes the time delay, $u \in \mathbb{R}$ is a process input and $y_p \in \mathbb{R}$ is a process output. The process input is bounded by $u_l \leq u(t) \leq u_u$. An optimal-control problem is in general defined as the design of a controller that asymptotically stabilizes a closed-loop system in such a way that the process output, $y_p(t)$, optimally follows a prescribed reference trajectory, $y_r(t)$, according to the given performance index. The solution of the classical optimal-control problem is difficult, and in this paper it is avoided by use of the moving-horizon control concept [14, 5, 4]. Furthermore, the system's nonlinearity presents an additional difficulty in terms of system modelling and control. This problem can be successfully solved by using a Wiener-type system that has a special structure that facilitates its application to model-based predictive control. The Wiener time-delayed system has the structure of a dynamic linear block

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ v(t) &= Cx(t - T_d) \end{aligned} \tag{2}$$

where $A \in \mathbb{R}^n \times \mathbb{R}^n$, $B \in \mathbb{R}^n$ and $C \in \mathbb{R}^n$. The variable $v(t) \in \mathbb{R}$ represents the intermediate variable that when connected in series with a static nonlinearity forms the model output

$$y(t) = h(v(t)), \tag{3}$$

where $h : \mathbb{R} \to \mathbb{R}$ denotes the static nonlinear mapping and $y \in \mathbb{R}$ is the process-model output. Furthermore, we assume the so-called undelayed linear system, the output of which forms the auxiliary model output containing no time delays:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ \overline{v}(t) &= Cx(t) \\ \overline{y}(t) &= h(\overline{v}(t)) \end{aligned} \tag{4}$$

The static nonlinearities in this paper are approximated by using CPWL functions [11] and fuzzy systems [1]. The stress will be on investigating the differences in the performance of closed-loop continuous-time model predictive control when the approximations are used individually in the model-output prediction.

## 2.1 CPWL approximation

The process-model output using the CPWL approximation is defined as

$$y_{mp}(t) = \hat{h}_{mp}(v(t)) = \Theta^T \Lambda(v(t)), \tag{5}$$

where $\Theta^T \in \mathbb{R}^{\sigma+1}$ and $\Lambda \in \mathbb{R}^{\sigma+1}$. Using the CPWL approximation, any nonlinear function $h$ can be uniquely represented by the segmentation of its input domain. Let us consider the segmentation into $\sigma$ segments by the parameters $\alpha_i$, with $\alpha_0 \leq \alpha_1 \leq \ldots \leq \alpha_\sigma$. In addition, the elements of the basis functions can be expressed as

$$\Lambda(v) = \begin{bmatrix} 1 \\ \frac{1}{2}\left(v - \alpha_0 + |v - \alpha_0|\right) \\ \vdots \\ \frac{1}{2}\left(v - \alpha_{\sigma-1} + |v - \alpha_{\sigma-1}|\right) \end{bmatrix} \tag{6}$$

and the vector of the parameters is defined as

$$\Theta^T = \left[\theta_0,\ \theta_1, \ldots, \theta_\sigma\right]. \tag{7}$$

The locations of the segments are chosen by clustering algorithms [18], and the vector of the parameters can be calculated using common least-square algorithms.

### 2.2 Fuzzy-system approximation

A fuzzy TS-type system in affine form with one antecedent variable and two consequent parameters is assumed. It can be given as a set of rules in the form

$$\mathbf{R}_{\mathrm{j}}: \ \textbf{if } x_p \textbf{ is } \mathbf{A}_j, \ \ \textbf{then } y_{mf} = \theta_{j,0} + \theta_{j,1} x_{c1}, \tag{8}$$

where $j = 1, \ldots, m$ is the number of fuzzy rules. The variable $x_p$ denotes the input or variable in premise, and the variable $y$ is the output of the model. The antecedent variable is connected with $m$ fuzzy sets $\mathbf{A}_j$, and each fuzzy set $\mathbf{A}_j$ ($j = 1, \ldots, m$) is associated with a real-valued function $\mu_{A_j}(x_p) : \mathbb{R} \rightarrow [0, 1]$, that produces a membership grade of the variable $x_p$ with respect to the fuzzy set $\mathbf{A}_j$. The consequent vector is denoted $x_c^T = [1, x_{c1}]$, and it implicitly represents an additional input to the fuzzy system. The system output is a linear combination of the consequent states. The system in (8) can be described in closed form

$$y_{mf} = \beta^T(x_p)\Theta_f x_c, \tag{9}$$

where the membership vector $\beta^T(x_p) = [\beta_1(x_p), \ldots, \beta_m(x_p)]$ is composed of normalized degrees of fulfilment

$$\beta_j(x_p) = \frac{\mu_{A_j}(x_p)}{\sum_{j=1}^{m} \mu_{A_j}(x_p)}, \ j = 1, \ldots, m, \tag{10}$$

and the matrix of fuzzy-model parameters

$$\Theta_f^T = \left[ \begin{array}{ccc} \theta_1 & \ldots & \theta_m \end{array} \right] \tag{11}$$

is composed of vectors of parameters in individual fuzzy domains:

$$\theta_j^T = [\, \theta_{j,0} \ \ \theta_{j,1} \,], \quad j = 1, \ldots, m \tag{12}$$

In this particular case the products of the parameter vectors and the consequent vectors, $\theta_j x_c$, form affine output functions. It is obvious that $\sum_{j=1}^{m} \beta_j(x_p) = 1$ irrespective of $x_p$ as long as the denominator of $\beta_j(x_p)$ in Eq. (10) is not equal to zero (which can easily be prevented by stretching the membership functions over the whole potential area of $x_p$).

Using the intermediate variable $v(t)$ as the antecedent variable $x_p$, the nonlinear output mapping can be written in closed form as

$$y_{mf}(t) = \hat{h}_{mf}(v(t)) = \beta^T(v(t))\Theta_f x_c(v(t)), \tag{13}$$

where $\beta^T \in \mathbb{R}^m$, $\Theta_f \in \mathbb{R}^m \times \mathbb{R}^2$ and $x_c \in \mathbb{R}^2$.

# 3 Nonlinear model-predictive control of Wiener-type time-delayed systems

In general the objective of a model-predictive control law is to drive the predicted future output of a system as close as possible to the future reference, subject to the input constraints. In the continuous-time framework this implies that the predictions of the reference and the process output must be either known or estimated. Let us define the reference model by the triple in state-space as $A_r$, $B_r$ and $C_r$ and denote the reference signal as $w(t)$. In the moving time frame the model-output prediction at time $\tau$ can be approximated by a truncated Maclaurin series expansion

$$y(t + \tau|t) = \Gamma^T(\tau)Y(t) \tag{14}$$

where the vectors $\Gamma$ and $Y$ are given by

$$\Gamma(\tau) = \left[1 \; \tau \ldots \frac{\tau^i}{i!} \ldots \frac{\tau^{n_y}}{n_y!}\right]^T, \tag{15}$$

$$Y(t) = \left[y(t) \; y^{[1]}(t) \ldots \; y^{[i]}(t) \; \ldots y^{[n_y]}(t)\right]^T, \tag{16}$$

with $Y \in \mathbb{R}^{n_y}$, $n_y$ is the output order, and $y^{[i]}(t)$ stands for the $i$th derivative of $y(t)$ with respect to $t$. Analogously, the reference-model output prediction can be defined as

$$y_r(t+\tau|t) = \Gamma^T(\tau) \cdot r \cdot w(t), \tag{17}$$

where the vector of the Markov parameters $r \in \mathbb{R}^{n_y+1}$ is defined as

$$r = \left[0 \;\; C_r B_r \;\; C_r A_r B_r \;\; \cdots \;\; C_r A_r^{n_y-1} B_r\right]^T. \tag{18}$$

Based on the idea of the Smith predictor, when dealing with systems containing pure time delays the future reference must be compared to the undelayed process output. The future control error should decrease according to the dynamics defined by the reference model

$$e(t+\tau) = \Gamma^T r \left(w(t) - \overline{y}_p(t)\right), \tag{19}$$

where $\overline{y}_p(t)$ is the output of the estimated undelayed process output. Since the undelayed process output is not available, it has to be estimated from the actual process output and the process model. We assume that the difference between the actual and the undelayed process outputs is equal to the difference between the delayed and the undelayed process-model outputs:

$$y_p(t) - \overline{y}_p(t) = y(t) - \overline{y}(t). \tag{20}$$

In this sense the undelayed process output $\overline{y}_p$ can be replaced by

$$\overline{y}_p(t) = y_p(t) - y(t) + \overline{y}(t). \tag{21}$$

The idea of the proposed continuous-time MPC, referring to the predictive functional control derivation [22], is based on a minimization of the difference between the future control error and the difference between the predicted model output at time horizon $\tau$, $\tau \in [0, T]$ and the current undelayed-model output:

$$\epsilon(t,\tau) = e(t+\tau) - (\overline{y}(t+\tau|t) - \overline{y}(t)) \tag{22}$$

The control law will be obtained by minimizing the cost function

$$V = \int_0^T \|\epsilon(t,\tau)\|^2 d\tau = \int_0^T \|e(t+\tau) - (\overline{y}(t+\tau|t) - \overline{y}(t))\|^2 d\tau. \tag{23}$$

Let us first investigate the model-output prediction (14) in the CPWL approximation case. The $i$th derivative of $y(t)$ is defined as

$$y_{mp}^{[i]}(t) = \Theta^T \frac{d\Lambda(v)}{dv} C A^i x(t) + \Theta^T \frac{d\Lambda(v)}{dv} \left[C A^{i-1} B \; \ldots \; C B\right] U(t), \tag{24}$$

where $U(t)$ stands for

$$U(t) = \left[u(t) \; u^{[1]}(t) \ldots u^{[i]}(t)\right]^T \tag{25}$$

and where

$$\frac{d\Lambda(v)}{dv} = \begin{bmatrix} 0 \\ \frac{1}{2}\left(1 + sign\left(v - \alpha_0\right)\right) \\ \vdots \\ \frac{1}{2}\left(1 + sign\left(v - \alpha_{\sigma-1}\right)\right) \end{bmatrix}. \tag{26}$$

Because all of the higher derivatives of the CPWL mapping with respect to $v$ are equal to 0 ($\frac{d^2\Lambda(v)}{dv^2} = \ldots = \frac{d^n\Lambda(v)}{dv^n} = 0$), all of the higher powers of $\dot{v}(t)$ are cancelled as well. This is, however, not the case when using the FS approximation. Differentiating (13) with respect to time, the first two derivatives will be as follows:

$$\dot{y}_{mf} = \frac{d\hat{h}_{mf}(v)}{dv} \cdot \dot{v}(t) \tag{27}$$

$$\ddot{y}_{mf} = \frac{d^2\hat{h}_{mf}(v)}{dv^2} \cdot (\dot{v}(t))^2 + \frac{d\hat{h}_{mf}(v)}{dv} \cdot \ddot{v}(t). \tag{28}$$

Since the first term on the right-hand side in (28) can be written as

$$\frac{d^2\hat{h}_{mf}(v)}{dv^2} = \frac{d^2\beta^T}{dv^2}\Theta_f x_c + 2\frac{d\beta^T}{dv}\Theta_f\frac{dx_c}{dv} + \beta^T\Theta_f\frac{d^2x_c}{dv^2}, \tag{29}$$

it is obvious that, even with the assumption that the second and higher derivatives of $\beta$ and $x_c$ are equal to 0, the first term cannot be canceled, and hence the analytical definition of the output prediction is too complex. For this reason, all the terms $(\dot{v}(t))^k,\ k \geq 2$ are assumed to be 0, and the prediction problem is reformulated to be very similar to the form in the CPWL case:

$$y_{mf}^{[i]}(t) = \frac{d\hat{h}_{mf}(v)}{dv} \cdot \frac{d^iv}{dt^i} = \frac{d\hat{h}_{mf}}{dv}\left(CA^ix(t) + \left[CA^{i-1}B\ \ldots\ CB\right]U(t)\right), \tag{30}$$

where

$$\frac{d\hat{h}_{mf}}{dv} = \frac{d\beta^T}{dv}\Theta_f x_c + \beta^T\Theta_f\frac{dx_c}{dv}. \tag{31}$$

Let us define the control order as follows.

**Definition 1** *The control order in the continuous-time predictive control is said to be $n_u$ if the following is valid: $u^{[n_u]}(t+\tau) \neq 0,\ \forall\ \tau \in [0, T]$ and $u^{[i]}(t+\tau) = 0,\ \forall\ i > n_u,\ \tau \in [0, T]$ where $u^{[n_u]}(t+\tau)$ stands for $n_u$th derivative of $u(t+\tau)$ with respect to $\tau$. The control order defines the allowable set, $\mathcal{U}$, of the optimal control input in the receding horizon frame, and hence imposes the constraints on $u(t+\tau)$.*

**Remark 1** *In this paper the output order $n_y$ and the control order $n_u$ are two design parameters. However, there are some limitations in the choice of $n_y$. If the relative order of a process is denoted $\rho$, $n_y$ should be at least of the same order as $n_u + \rho$ if the $n_u$th derivative of the control signal is to appear in the prediction, i.e., $n_y \geq n_u + \rho$.*

The control vector $U(t)$ of the $n_u$th order is then defined as

$$U(t) = \left[u(t)\ u^{[1]}(t) \ldots u^{[n_u]}(t)\right]^T. \tag{32}$$

Combining equations (14)-(16) with (24) and (30), the predictions of the model outputs $y_{mp}(t+\tau|t)$ and $y_{mf}(t+\tau|t)$ at time $\tau$ are defined as

$$y(t+\tau|t) = \Gamma^T\left[Py(t) + Q(v)x(t) + H(v)U(t)\right], \tag{33}$$

where $P \in \mathbb{R}^{n_y+1}$ is

$$P = [1\;0\;\ldots\;0]^T. \tag{34}$$

The matrices $Q \in \mathbb{R}^{n_y+1} \times \mathbb{R}^n$ and $H \in \mathbb{R}^{n_y+1} \times \mathbb{R}^{n_u+1}$ are calculated differently for each of the approximation cases. In the CPWL case we can write $Q(v) = q_p(v)K_q$ and $H(v) = q_p(v)K_h$, and analogously in the FS case $Q(v) = q_f(v)K_q$ and $H(v) = q_f(v)K_h$, where $q_p$, $q_v$, $K_q$ and $K_h$ are defined as

$$q_p(v) = \Theta^T \frac{d\Lambda(v)}{dv}, \tag{35}$$

$$q_f(v) = \frac{d\beta^T(v)}{dv}\Theta_f x_c(v) + \beta^T(v)\Theta_f \frac{dx_c(v)}{dv}, \tag{36}$$

$$K_q = \left[0\;CA\;CA^2\;\ldots\;CA^{n_y}\right]^T, \tag{37}$$

and

$$K_h = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ CA^{n_y-1}B & CA^{n_y-2}B & \cdots & CA^{n_y-1-n_u}B \end{bmatrix}. \tag{38}$$

Given the prediction of the process-model output in (33), the cost function (23) is

$$V(U,t) = \int_0^T \left(\left(w - \overline{y}_p\right)^T r^T - U^T H^T - x^T Q^T\right) \Gamma\Gamma^T \left(r\left(w - \overline{y}_p\right) - HU - Qx\right) d\tau \tag{39}$$

Notice that, taking into account the calculation in (33), the product $\Gamma^T P\overline{y}(t)$ is equal to $\overline{y}(t)$, and hence cancels the last term of (22). The minimization of the cost function results in the continuous-time model-predictive control law

$$\frac{\partial V}{\partial U} = -2H^T \left[\int_0^T \Gamma\Gamma^T r\left(w - \overline{y}_p\right) d\tau - \int_0^T \left(\Gamma\Gamma^T HU + \Gamma\Gamma^T Qx\right) d\tau\right] = 0. \tag{40}$$

Let us define the matrix $\overline{\Gamma} \in \mathbb{R}^{n_y+1} \times \mathbb{R}^{n_y+1}$ as

$$\overline{\Gamma} = \int_0^T \Gamma\Gamma^T d\tau. \tag{41}$$

Given that the general term of the matrix $\Gamma\Gamma^T$ is $T^{i-1+j-1}/((i-1)!(j-1)!)$, equation (41) can be rewritten as

$$\overline{\Gamma} = \begin{bmatrix} \gamma_{(1,1)} & \cdots & \gamma_{(1,n_y+1)} \\ \vdots & \ddots & \vdots \\ \gamma_{(n_y+1,1)} & \cdots & \gamma_{(n_y+1,n_y+1)} \end{bmatrix}, \tag{42}$$

where

$$\gamma_{(i,j)} = \frac{1}{(i+j-1)(i-1)!(j-1)!} T^{i+j-1} \tag{43}$$

for every $i, j = 1, \ldots, n_y + 1$. Equation (40) is then reformulated as

$$\frac{\partial V}{\partial U} = -2H^T\overline{\Gamma}\left[r\left(w - \overline{y}_p\right) - HU - Qx\right] = 0 \tag{44}$$

and, using the substitution from (21), the control vector becomes

$$U = \left(H^T\overline{\Gamma}H\right)^{-1} H^T\overline{\Gamma}\left[r(w - y_p + y - \overline{y}) - Qx\right]. \tag{45}$$

At this point we return to the separate notation for the CPWL and FS cases and employ the notation in (35)-(38). Let us define the first row of the matrix $\left(H(v)^T\overline{\Gamma}H(v)\right)^{-1}H(v)^T\overline{\Gamma} \in \mathbb{R}^{n_u+1} \times \mathbb{R}^{n_y+1}$ in the CPWL case as $\kappa_p(v)$ and analogously in the FS case as $\kappa_f(v)$. Now the control law of the nonlinear Wiener-type model-predictive control is given by

$$u(t) = \kappa_p(v)r(w - y_p + y_{mp} - \overline{y}_{mp}) - \kappa_p(v)Q_p(v)x \tag{46}$$

for the CPWL approximation case and

$$u(t) = \kappa_f(v)r(w - y_p + y_{mf} - \overline{y}_{mf}) - \kappa_f(v)Q_f(v)x \tag{47}$$

for the FS case.

# 4 Simulation example

The proposed method was tested on a third-order linear time-delayed system

$$G_p(s) = \frac{v(s)}{u(s)} = \frac{1.2}{(s+0.2)(s+2)(s+3)} \cdot e^{-5s} \tag{48}$$

with a static, nonlinear output mapping

$$y_p(t) = 4.2 \cdot \left(\arctan\left[10(v(t) - 0.5)\right] + \pi/2\right). \tag{49}$$

The control-design parameters were chosen as follows: $n_u = 1$, $n_y = 4$, $T = 0.5$ s, $A_r = -1/3$, $B_r = 1$, and $C_r = 1/3$. The choice of $n_y$ and $T$ will be discussed later. The process input was assumed to be bounded by the interval $0 \leq u(t) \leq 1$, hence the intermediate-variable range was $[0, 1]$. Even though the input bounds are not considered in the control-law derivation, bounding the controller output implicitly bounds the model-output prediction and the parameters for the control-action calculation. For the CPWL the intermediate-variable range was further divided into five non-equidistantly spread segments. The segment-parameter positions $\alpha_i,\ i = 1, 2, \ldots, 4$ were calculated using a c-means clustering method [1]. The initial parameter $\alpha_0$ was set to 0 because otherwise the derivative of $\hat{h}_{mp}$ in the interval $[0\ \alpha_1]$ would be 0. However, the parameter set is in this way augmented by 1. The optimization of the parameters $\Theta$ was carried out using linear programming. The resulting parameter vectors are given by

$$\begin{aligned}\alpha &= [0,\ 0.176,\ 0.437,\ 0.563,\ 0.824],\\ \Theta &= [0.919,\ 0.339,\ 9.895,\ 36.441,\ -36.420,\ -9.923].\end{aligned}$$

In the FS case we assumed four triangular membership functions and linear output functions, i.e., $x_c = [1, v(t)]^T$ and $\Theta_f \in \mathbb{R}^4 \times \mathbb{R}^2$. The membership function apexes were at the same positions as in the CPWL case, i.e., $\alpha_i,\ i = 1, 2, \ldots, 4$. Therefore, after completing the linear-programming optimization, the fuzzy-parameter matrix yielded

$$\Theta_f^T = \begin{bmatrix} 0.778 & -1.715 & 1.645 & 9.537 \\ 2.877 & 13.241 & 13.276 & 2.880 \end{bmatrix}.$$

The resulting approximations $\hat{h}_{mp}(v)$ and $\hat{h}_{mf}(v)$ are compared to the actual mapping $\hat{h}(v)$ in Figure 1.

One way to compare the effects due to the approximation choice is to investigate the behaviour of the open-loop model predictions in both cases. Fig. 2 shows the linear-model output prediction dependent on $n_y$. It is clear that increasing the value of $n_y$ improves the prediction accuracy, and hence the prediction horizon $T$ can easily be increased. However, in the nonlinear-model case

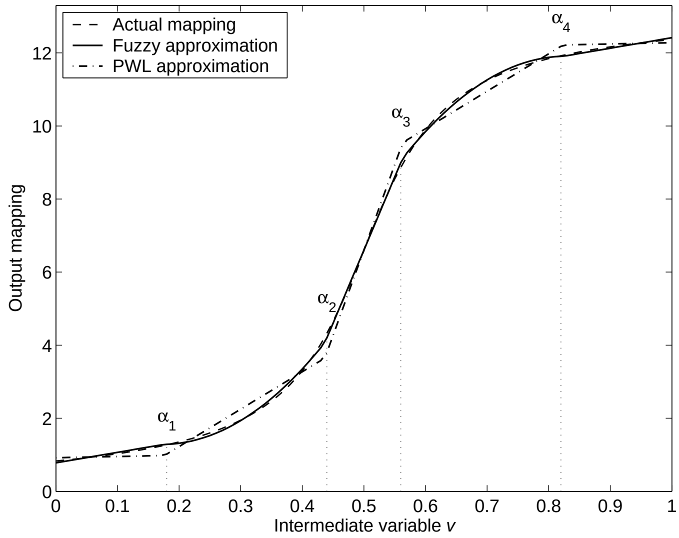


Figure 1: Comparison of the CPWL and FS approximations of the output mapping using four clusters

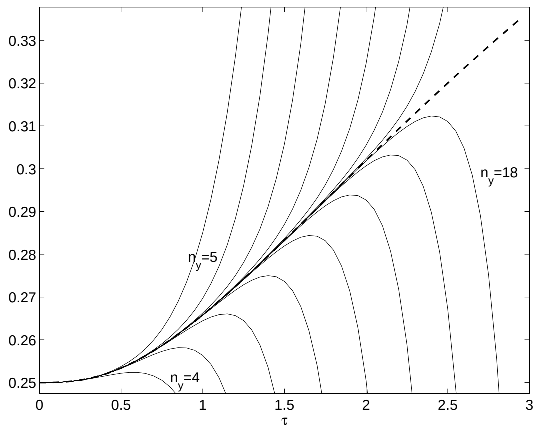


Figure 2: Linear-model output prediction in terms of $n_y$

some of the prediction error can be attributed to the approximation accuracy. Figure 3 shows the benefit of choosing the FS approximation rather than the CPWL approximation. In all three regions the FS prediction gives better results, even though in the FS approximation procedure the higher derivatives of $v$ were left out. Nevertheless, in terms of the horizon choice there is little or no change from the linear-model case - the choice of $n_y$ is still dictated by the choice of $T$. In our case if we wanted to make a fair comparison, the value of $T$ had to be chosen so that it was low enough for the model-output predictions to be fairly accurate. When choosing $T = 0.5$, a fourth-order output prediction ($n_y = 4$) was sufficient.

A closed-loop experiment with a series of step signals as the reference signal was conducted. The results, presented in Figure 4, imply that the process output successfully follows the reference-model output for the whole operating region in both approximation cases, and, at the same time, does not suffer from the process time delay. However, it is clear that in the FS case the overall performance is much better. In the CPWL case the reference tracking depends on the operating point much more than in the opposite case, and in some regions the system output is subject to oscillations. The corresponding input variables are shown in Figure 5. In the CPWL case the input is much more oscillatory. Also notice the impulse-like behaviour of the control signals in the time

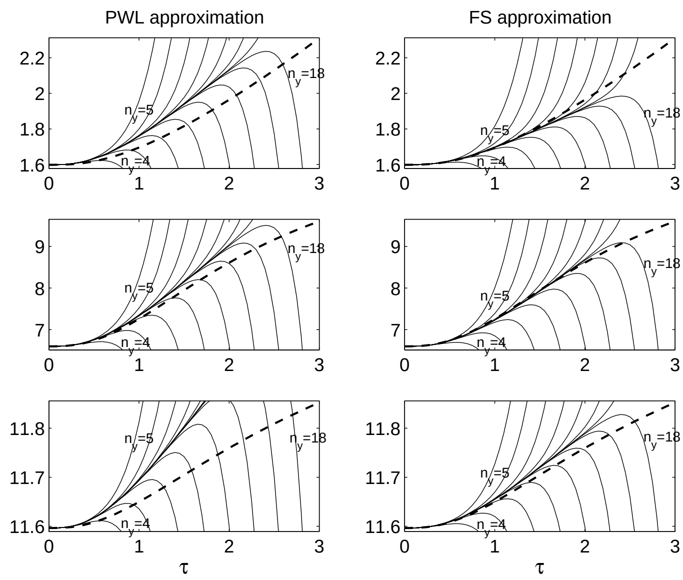


Figure 3: Comparison of the CPWL and FS nonlinear-model predictions in terms of $n_y$

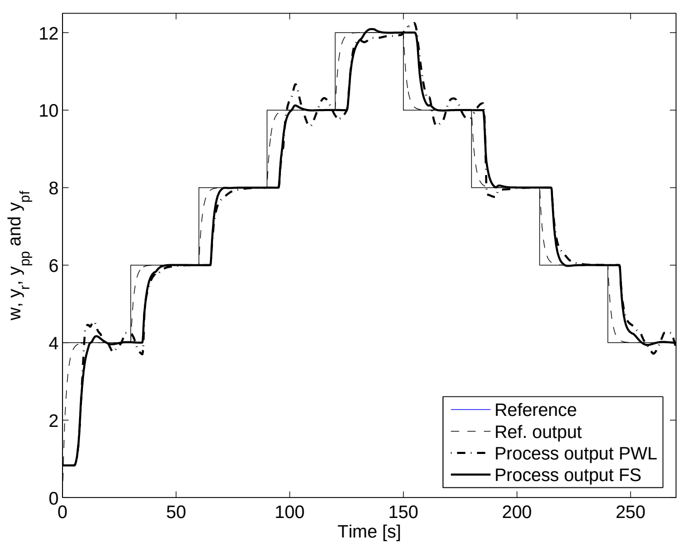


Figure 4: Comparison of the closed-loop-experiment results for both approximation cases

after the step reference changes - this is because a third-order system was made to closely follow a first-order reference model.

# 5 Conclusion

A novel method of continuous-time model-predictive control for nonlinear time-delayed systems was presented. In the derivation procedure the method implicitly incorporates two different static nonlinear-mapping approximations - using continuous piece-wise linear functions and a fuzzy system - and the solution for tackling system time delays. It was shown for the case of a third-order system with an arcus-tangent output mapping and a considerable pure time delay that the proposed closed-loop system in both cases exhibits quality control; however, the performance in the FS case is clearly better due to a better model-output prediction. The proposed method is

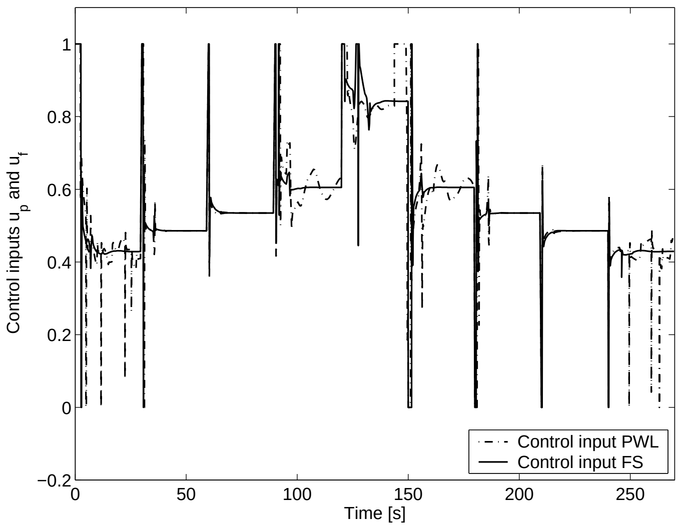


Figure 5: Control signals in the experiment with our proposed method

thus very appropriate for chemical processes that can be described by a time-delayed Wiener-type system and where high-quality control is desired.

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